

DEU ALGEBRA DAY

JULY 4, 2022

SPEAKERS

- Merve ALGI (AKU)
- Victor BLASCO (DEU)
- Mücahit BOZKURT (ÇU)
- Nefise CEZAYİRLİOĞLU (EU)
- Ayşenur DİNÇER (AKU)
- Canan ÖZEREN (DEU)
- İrem YILDIZ (DEU)

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Abstract Book

DEU ALGEBRA DAY

July 4, 2022

Buca, İzmir, Türkiye

<https://math.deu.edu.tr/deu-algebra-day-july-4-2022/>

Organized by

Dokuz Eylül University

İzmir, Türkiye

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Preface

Dear Participants,

“DEU ALGEBRA DAY, July 4, 2022” is an event organized by the DEU algebra group. The goal of DEU ALGEBRA DAY is to bring graduate students together who mainly focuses on algebra, providing an opportunity for young researchers to enhance their presentation skills, connecting them with important research directions and help expanding their network of research mentors.

In this booklet, you can find all abstracts and details of the event.

Yours Sincerely,
Organizing Committee

DEU ALGEBRA DAY, July 4, 2022

Program

DEU ALGEBRA DAY, July 4, 2022 Program		
04.07.2022		
Time	Talks	B256
09:30–10:00	<i>Opening</i>	
10:00–10:25	Merve Algı	Chair: Noyan Er
10:25–10:50	Mücahit Bozkurt	Chair: Noyan Er
10:50–11:35	<i>Break</i>	
11:35–12:00	Ayşenur Dinçer	Chair: Fatma Kaynarca
12:00–13:30	<i>Lunch</i>	
13:30–13:55	Victor Blasco	Chair: Çağrı Demir
13:55–14:20	İrem Yıldız	Chair: Çağrı Demir
14:20–15:00	<i>Break</i>	
15:00–15:25	Canan Özeren	Chair: A. Tuğba Güroğlu
15:25–15:50	Nefise Cezayirlioğlu	Chair: A. Tuğba Güroğlu

Talks

Gröbner Basis

Merve Algi

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Gröbner bases were introduced in 1965 by Bruno Buchberger in his Ph.D. thesis and he named them after his advisor Wolfgang Gröbner (1899-1980). The basic idea behind the theory can be described as a generalization of the theory of polynomials in one variable. In the polynomial ring $k[x]$ where k is a field, any ideal I can be generated by a single element, namely the greatest common divisor of the elements of I . Given any set of generators $\{f_1, f_2, \dots, f_s\} \subseteq k[x]$ for I , one can compute (using the Euclidean Algorithm) a single polynomial $d = \gcd(f_1, f_2, \dots, f_s)$ such that $I = \langle f_1, f_2, \dots, f_s \rangle = \langle d \rangle$. Then a polynomial $f \in k[x]$ is in I if and only if the remainder of the division of f by d is zero. Gröbner bases are the analog of greatest common divisors in the multivariate case in the following sense. A Gröbner basis for an ideal $I \subseteq k[x_1, x_2, \dots, x_s]$ generates I and a polynomial $f \in k[x_1, x_2, \dots, x_s]$ is in I if and only if the remainder of the division of f by the polynomials in the Gröbner basis is zero (the appropriate concept of division and ordering is a central aspect of the theory). Firstly, we will give the basic introduction to the concept of a Gröbner basis and show how to compute it using Buchberger's Algorithm. Then we will extend the theory of Gröbner bases to submodules of a finitely generated free module over a polynomial ring. Thus we will show that the syzygy modules of finitely generated modules can be computed.

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TORSION-FREE MODULES

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Let R be an integral domain and M be a right R -module. M is called torsion-free module if $xr = 0$ implies that either $x = 0$ or $r = 0$ for $x \in M$ and $r \in R$. In [1], Hattori defined and examined torsion-free modules over a noncommutative ring inspired by the homological property of torsion-free modules over an integral domain. Let R be a noncommutative ring and X a right R -module. X is said to be torsion-free module if $Tor_1(X, R/Ra) = 0$ for all $a \in R$. Torsion-free modules are intimately related to relatively divisible (RD) modules. A left R -module C is torsion-free if and only if every exact sequence $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$ is RD -exact sequence. Let M be a right R -module and N be a left R -module. If, for every exact sequence of left R -modules $0 \rightarrow H \rightarrow F \rightarrow N \rightarrow 0$, $0 \rightarrow M \otimes H \rightarrow M \otimes F \rightarrow M \otimes N \rightarrow 0$ is exact, then M is said to be an N -subflat. If M is N -subflat module, all family of N modules is called the subflat domain of M and it is denoted by $\mathfrak{F}^{-1}(M)$. A right R -module M is flat if and only if $\mathfrak{F}^{-1}(M) = R - Mod$. If M is RD -flat, $\mathfrak{F}^{-1}(M) = R - Mod$ consists of all torsion-free modules. If $\mathfrak{F}^{-1}(M)$ is a class of torsion-free modules, we say that RD -module M is test module, shortly tf -module, for torsion-free modules. In this work, we investigate some properties of subflat domains, a new characterization of torsion-free rings via subflat domains, and the existence of test modules for torsion-freeness.

This research was supported by the Scientific and Technological Research Council of Türkiye. Project number: 119F176.

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Auslander-Reiten Quivers of Type \mathbb{A}_n

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Since the late 1960s, the representation theory started growing fast owing to the introduction of almost split sequences by Maurice Auslander and Idun Reiten, and of quivers and their representations by Peter Gabriel. The use of quivers in the representation theory of finite-dimensional algebras gives us the possibility to visualize the modules of a given algebra very concretely as a collection of matrices, each of which is associated to an arrow in a certain diagram in the quiver. The main tool for describing the representation theory of a finite-dimensional algebra is the Auslander–Reiten quiver, which gives explicit information about the modules as well as the morphisms between them in a most convenient way. The Auslander–Reiten quivers provide a threefold information about the representation theory of the quiver, namely the indecomposable representations, the irreducible morphisms, and the almost split sequences (these in turn should be thought of the building blocks of arbitrary representations, morphisms, and short exact sequences, respectively). We present several methods to compute the Auslander–Reiten quiver of a quiver type \mathbb{A}_n . The first method, the knitting algorithm, is a recursive procedure which owes its name to the fact that it produces one mesh after the other. The second method is to compute the orbits under the Auslander–Reiten translation τ . The third method is a triangulation of a polygon as a geometric way. We will explain “Why does one draw an Auslander-Reiten quiver?”. Finally, we will give a brief history of representation theory highlighting the importance of Auslander-Reiten quivers.

References

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Dualities in Artin Algebras

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In this talk, after introducing the categorical notion of duality in terms of contravariant functors and looking at some examples, we will show the existence of a duality between the category of finitely generated modules over an Artin Algebra and over its opposite algebra.

References

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Thin Quivers Representations

İrem Yıldız

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In this talk we will see some types of quivers and basic properties of the representations over them. A representation is called thin if all the vector spaces have dimension less than two. We will prove that all indecomposable representations over a finite connected quiver Q are thin if and only if Q is of type A_n .

References

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coGalois Groups of Quivers

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Given a module M and an \mathcal{F} -cover $\varphi : F \rightarrow M$, the *coGalois group* of φ is the subgroup $G(\varphi)$ of $\text{Aut}(F)$, consisting of all automorphisms $g : F \rightarrow F$ such that $\varphi \circ g = \varphi$. In this talk, we will give the motivation for our study of coGalois groups associated to covers in the category of representations of quivers.

References

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On a Certain Functional Identity Involving Inverses in Division Rings and Local Rings

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Let D be a division ring, σ an automorphism of D and $u \in D$ a fixed element. We determine the forms of additive maps $f, g : D \rightarrow D$ satisfying the identity

$$f(x)x^{-1} + \sigma(x)g(x^{-1}) = u$$

for every nonzero $x \in D$. We show that f and g are both generalized σ -derivations of D except possibly when σ is the identity automorphism and D is an imperfect field of characteristic two. We further study the same rational identity in the context of local ring under some mild assumptions.

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