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# DRBEM SOLUTIONS OF REGULARLY AND SINGULARLY PERTURBED MAGNETOHYDRODYNAMIC DUCT FLOW PROBLEMS

Sinem Arslan Ölçer<sup>1</sup>, Münevver Tezer-Sezgin<sup>2</sup>

<sup>1,2</sup> Department of Mathematics, Middle East Technical University, Ankara, Turkey

<sup>1</sup> arsinem@metu.edu.tr, <sup>2</sup> munt@metu.edu.tr

## Abstract:

This study investigates the dual reciprocity boundary element method (DRBEM) solutions of magnetohydrodynamic (MHD) flow problems in rectangular ducts, which are either regularly or singularly perturbed, under the influence of a magnetic field and general boundary conditions. The steady, laminar and fully-developed MHD flow of an incompressible, viscous, and electrically conducting fluid in a long channel of square cross-section (duct) is driven by a pressure gradient. The governing flow equations are convection-diffusion type and coupled in terms of the velocity and induced magnetic field. The aim is to construct perturbation theory on these coupled equations based on the Hartmann number ( $Ha$ ). For small  $Ha$ , the regular perturbation method is employed using power series expansions of the unknowns in terms of the perturbation parameter. For large  $Ha$ , the singular perturbation method is applied by generating a Shishkin mesh adapted to boundary layer thicknesses, where capturing numerical solutions becomes challenging due to convection dominance. The DRBEM is used to discretize the coupled MHD flow equations with the fundamental solution of Laplace equation, transforming them into boundary integral equations. All the terms other than Laplacian are approximated by radial basis functions. The resulting discretized equations are solved by Gauss-elimination. The most important contribution of this study is that, it is the first DRBEM application to the singularly perturbed two-dimensional MHD duct flow problems, in which parallel computation is also adopted for reaching higher values of  $Ha$ . The flow and the current are simulated in terms of contour plots and level curves and their expected behaviors are well observed.

**Keywords:** Magnetohydrodynamics, Dual Reciprocity Boundary Element Method, Singular perturbation, Shishkin mesh, Regular perturbation

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