

Algebra in Coding Theory: A Focus on Algebraic Geometric Codes

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Algebraic geometric codes are distinguished by their strong interplay between algebraic and geometric structures. Understanding these connections and establishing bounds for key parameters such as the dimension and minimum distance are among the central problems in coding theory. Algebraic invariants provide powerful tools for tackling these challenges. In particular, the dimension of Weighted Projective Reed–Muller codes is known to be closely related to the Hilbert function, illustrating the deep link between algebra and coding theory. This talk aims to highlight the significance of algebraic methods in coding theory, focusing especially on such code families, and to outline some of the open problems that continue to attract research interest in this area.

Keywords. Algebraic Geometric Codes, Coding Theory, Weighted Projective Spaces, Main Parameters, Algebraic Invariants.

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